

Adjoint-interpolation using Atlas

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Met Office Motivation for adjoint interpolation

Initial issue here is that we want an atlas-derived background error covariance that can deal with both EndGame (staggered longitude-latitude grids) and LFRic (cubed-sphere grid).

In both systems we are using a ECMWF trans library to do the spectral covariances. This package requires data to be on a Gaussian latitude grid. So we need interpolation from gaussian grid to model grid (and adjoint).

| | | | | |
|---|---|---|--|---|
| (Spatial part of B matrix not including variable changes) | = | Interpolation from Gaussian latitude grid to model grid | Spectral B on Gaussian latitude grid | Adjoint of interpolation from Gaussian latitude grid to model grid. |
|---|---|---|--|---|

We want spectral B to work for any global model grid

Motivation for adjoint interpolation (2)

Bigger issue:

Could interpolation schemes in Atlas be more widely used in JEDI?

Adjoint of interpolation schemes could help that happen.

Could code be leveraged to for efficient interpolation to observations?

Basic procedure.

| Forward | Adjoint |
|----------|-----------------|
| $y = Ax$ | $x = x + A^* y$ |
| $x = x$ | $y = 0$ |

When A is linear operator it can be represented as a matrix.

When x and y are real the adjoint operator A^* can be represented as a transpose of matrix A ie A^T .

Met Office Atlas interpolation

Many of Atlas's interpolation schemes store interpolation weights in eckit `sparse_matrix` container. (Olly noticed this!)

`eckit sparse_matrix` has a `transpose` method for constructing a transpose matrix. So we have all that we need. 😊 (I added the adjoint of the halo-exchange previously).

Met Office Sparse Matrix Formulation

Row major format.

- *outer_* = start of rows
- *inner_* = column indices
- *data_* = matrix entries

$$A = \begin{pmatrix} 11 & 12 \\ 21 & 22 \end{pmatrix}$$

outer_ = [0, 2, 4]

inner_ = [0, 1, 0, 1]

data_ = [11, 12, 21, 22]

Note that:

- Atlas's backend to this can use OpenMP
- Calculating on the fly the transpose matrix in this format can lead to race-conditions when using OpenMP.
- That is why we store the transpose matrix in the constructor and treat it as a “new” matrix.

Met Office A question of interfaces

Originally implemented as on slide 4. It is in ecmwf atlas develop branch. However ...

In oops we tend to not zero y in adjoint code ... instead we set it as constant (in C++ sense) to avoid unnecessary calculations. This is only a problem if that adjoint variable is reused.

There is a quick PR (in review) to ecmwf atlas develop to be more consistent with how we do things in oops.

Testing:

Adjoint interpolation test are done for interpolation from octahedral grid O32 to octahedral grid O64 for double and floating precision for linear schemes that use linear and cubic basis functions.

Adjoint tests pass $\langle AA^T x, x \rangle = \langle Ax, Ax \rangle$ with relative tolerance

(for floating numbers $4e-6$) and

(for double precision $2e-14$).

Met Office Summary and Additional work

- We have implemented the capability for adjoint interpolation in atlas for any atlas interpolation scheme that is represented by a sparse matrix.

Additional work (taster of other work done by me and Olly)

- Olly has implemented a cubed-sphere mesh generator in atlas.
- Marek has done single PE interpolation on a cubed-sphere to arbitrary set of points taking advantage of cubed-sphere structures. (not in atlas develop and further work needed on edges of the cubed-sphere and using the cubed-sphere mesh generator.)
- Spectral part of um-jedi B broadly compares with operational B at N320 resolution.