“static” B in SOCA

- First implementation in ~08/2018 (second B-matrix code sprint)
- Designed to be fast, simple but not necessarily great!

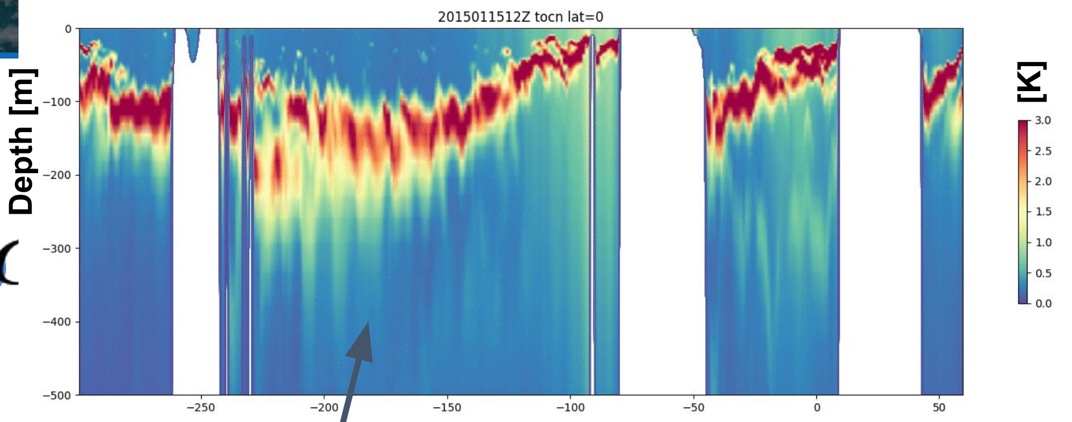
$$B = K F_h^{\frac{1}{2}} D_p D_f C_v^{\frac{1}{2}} C_h C_v^{\frac{1}{2}T} D_f D_p F_h^{\frac{1}{2}T} K^T$$

❖ Implemented in soca::ErrorCovariance
❖ 1 Fortran BUMP object per domain (ocean, sea ice, wave)
❖ Same correlation operators for all levels/variables. SocaError in Trait.h
❖ Decorrelation length:
  ➢ Ocean: scaled to the Rossby radius of deformation
  ➢ Sea ice: fixed scale
  ➢ Wave: “ “

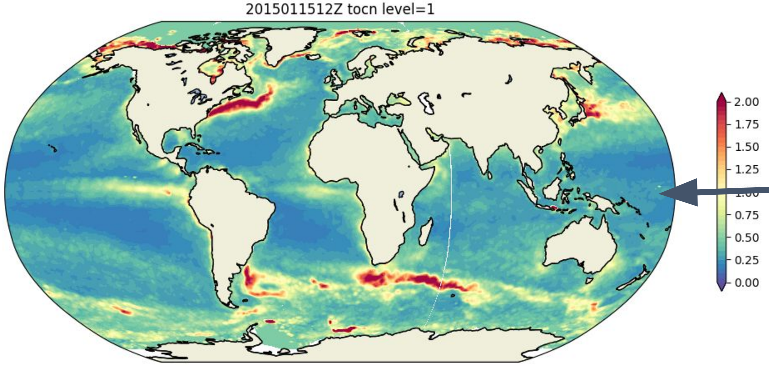
- Replace **$C_v^{½}$ C_h $C_v^{½}$** with 3D bump correlation operator derived from S2S re-forecasts

Parametric **B**

Equatorial Temperature background error



$$B = K F_h^{\frac{1}{2}} \underbrace{D_p D_f}$$



Parametric standard deviation of the background error

Ocean:

- Temperature
  - Based on dT/dz Below mixed layer
  - Climatology of SST obs - background within mixed layer (Hybrid-GODAS, Travis Sluka)
- Unbalanced Salinity, currents

Sea ice:

- Ice concentration, Ice thickness, snow depth

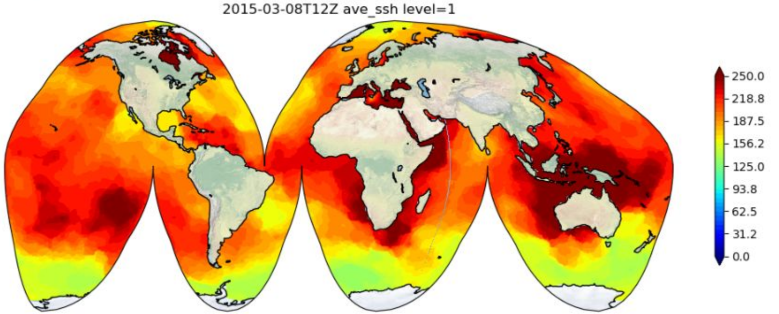
Wave

- Significant wave height

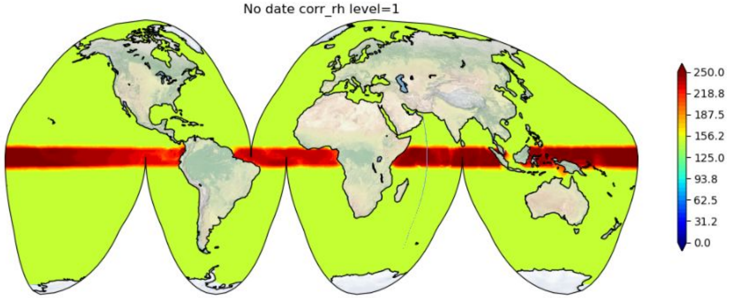
Ensemble based **B**

- Physically based balance operators
- Background error amplitude based on the vertical background temperature gradient and surface climatology of SST OMB's
- BUMP 3D correlation operator (Benjamin Menetrier):
    - Ensemble based for scales
    - Differ for each model levels and variables

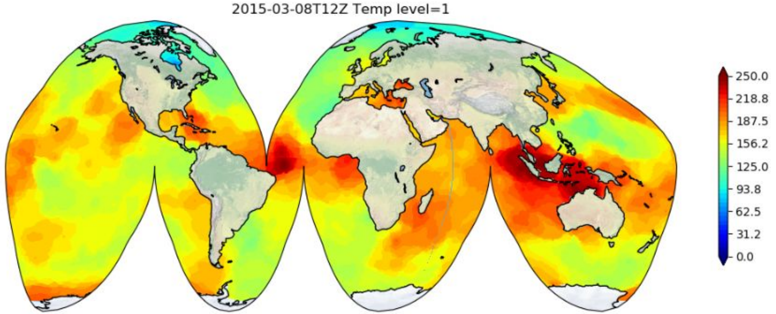
Estimated by BUMP for SSH:



Rossby radius-based length-scale (current **SOCA** B):



Estimated by BUMP for SST:

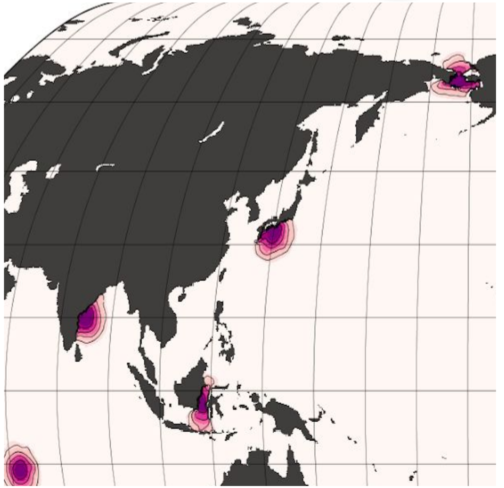


More below

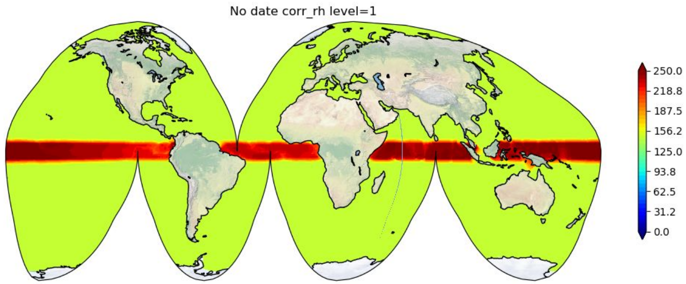
Default static B in SOCA

- First implementation in ~08/2018 (second B-matrix code sprint)
- A fairly standard parametric covariance model for the ocean

$$B = K F_h^{\frac{1}{2}} D_p D_f C_v^{\frac{1}{2}} C_h C_v^{\frac{1}{2}T} D_f D_p F_h^{\frac{1}{2}T} K^T$$



- ❖ 1 BUMP object per domain (ocean, sea ice, wave), same correlation operators for all levels/variables. SocaError in Trait.h
- ❖ Decorrelation length:
  - ➢ Ocean: scaled to the Rossby radius of deformation
  - ➢ Sea ice: fixed scale
  - ➢ Wave: “ “



BUMP C_h operator is similar to a diffusion operator at a fraction of the computational cost

Parametric **B**

$$B = K\underbrace{F_h^{\frac{1}{2}}}_{}\underbrace{D_p D_f}_{}C_v^{\frac{1}{2}} C_h C_v^{\frac{1}{2}^T} D_f D_p F_h^{\frac{1}{2}^T} K^T$$

Vertical convolution

Parametric standard deviation of the background error

Ocean:

- Temperature
  - Based on dT/dz Below mixed layer
  - Climatology of SST obs - background within mixed layer (Hybrid-GODAS, Travis Sluka)
- Unbalanced Salinity, currents

Sea ice:

- Ice concentration
- Ice thickness

Horizontal filter

Parametric **B**

$$\boldsymbol{B} = \boldsymbol{K} \boldsymbol{F}_h^{\frac{1}{2}} \underbrace{\boldsymbol{D}_p \boldsymbol{D}_f} \boldsymbol{C}_v^{\frac{1}{2}} \boldsymbol{C}_h \boldsymbol{C}_v^{\frac{1}{2}^T} \boldsymbol{D}_f \boldsymbol{D}_p \boldsymbol{F}_h^{\frac{1}{2}^T} \boldsymbol{K}^T$$

Weaver et al, 2006

$$\boldsymbol{K} = \begin{bmatrix} I & 0 & 0 & 0 \\ K_{ST} & I & 0 & 0 \\ K_{\eta T} & K_{\eta S} & I & 0 \\ K_{cT} & 0 & 0 & I \end{bmatrix}$$

$$\delta S_B = \frac{\partial S}{\partial T} \delta T$$

Troccoli and Haines, 1999

$$\delta \eta_B = -\int_{Bottom}^{0} \frac{\delta \rho(T, S, z)}{\rho_0} dz$$

Cooper and Haines, 1996

$$\delta c_B = \frac{\partial c}{\partial T} \delta T$$

Parametric **B**

$$\boldsymbol{B} = \boldsymbol{K} \boldsymbol{F}_h^{\frac{1}{2}} \underbrace{\boldsymbol{D}_p \boldsymbol{D}_f} \boldsymbol{C}_v^{\frac{1}{2}} \boldsymbol{C}_h {\boldsymbol{C}_v^{\frac{1}{2}}}^T \boldsymbol{D}$$

Weaver et al, 2006

$$\boldsymbol{K} = \begin{bmatrix} I & 0 & 0 & 0 \\ K_{ST} & I & 0 & 0 \\ K_{\eta T} & K_{\eta S} & I & 0 \\ K_{cT} & 0 & 0 & I \end{bmatrix}$$

$$\delta S_B = \frac{\partial S}{\partial T} \delta T$$

[Troccoli and Haines, 1999]()

$$\delta \eta_B = -\int_{Bottom}^{0} \frac{\delta \rho(T, S, z)}{\rho_0} dz$$

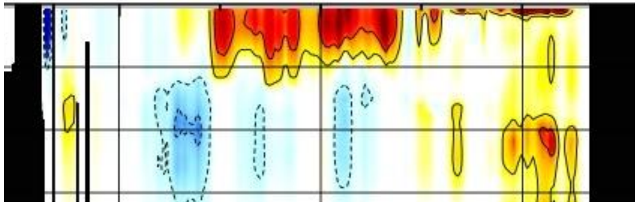
[Cooper and Haines, 1996]()

$$\delta c_B = \frac{\partial c}{\partial T} \delta T$$

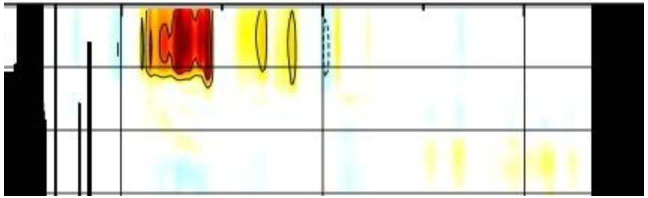
Application example: Altimeter assimilation

Multivariate increment for T and S using balance operators in the B-matrix

Temperature increment at 0N



Salinity increment at 0N



Parametric **B**: Use cases

- 1 deg global for **reanalysis** & ocean **monitoring**
- ¼ deg global for **S2S initialization**: Native res h(x), ½ deg B
- **HAFS initialization**: Regional HAT10 ¼ and 1/12 resolution
- **Gulf Of Mexico** (Travis?)
- Similar **B** used at ECMWF, UKMO, Navy/NOAA (NCODA), ...

Static Covariance Modeling in soca

- Access to SABER/BUMP

- Future plan: Ensemble (climatological or otherwise) based **B** (in progress, kind of)
  - **C** derived from an ensemble but parametric **D**
  - **D** derived from ensemble of ($\mathbf{K}^{-1}\mathbf{X}$)

bump for soca: Benjamin's Ocean predict presentation

Ensemble based **B** *("in progress")*

- Physically based balance operators
- Background error amplitude based on the vertical background temperature gradient and surface climatology of SST OMB's
- BUMP 3D correlation operator (Benjamin Menetrier):
  - Ensemble based for scales
  - Differ for each model levels and variables

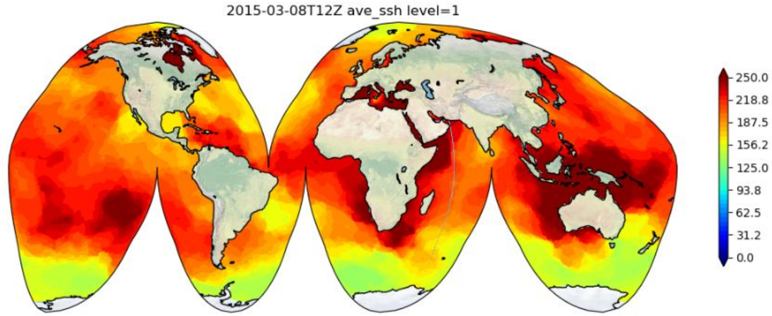
$$B = K F_h^{\frac{1}{2}} D_p D_f C_v^{\frac{1}{2}} C_h C_v^{\frac{1}{2}T} D_f D_p F_h^{\frac{1}{2}T} K^T$$

Replaced by a 3D ensemble based correlation operator from BUMP

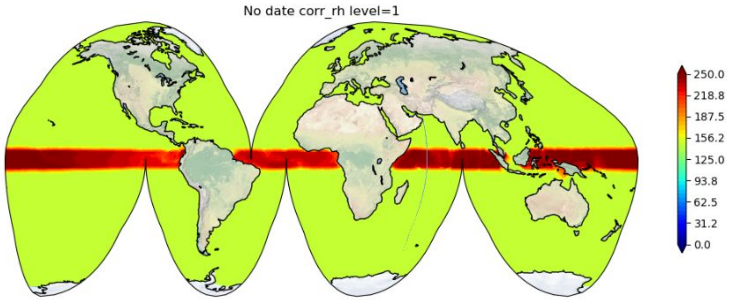
Ensemble based **B**

- Physically based balance operators
- Background error amplitude based on the vertical background temperature gradient and surface climatology of SST OMB's
- BUMP 3D correlation operator (Benjamin Menetrier):
  - Ensemble based for scales
  - Differ for each model levels and variables

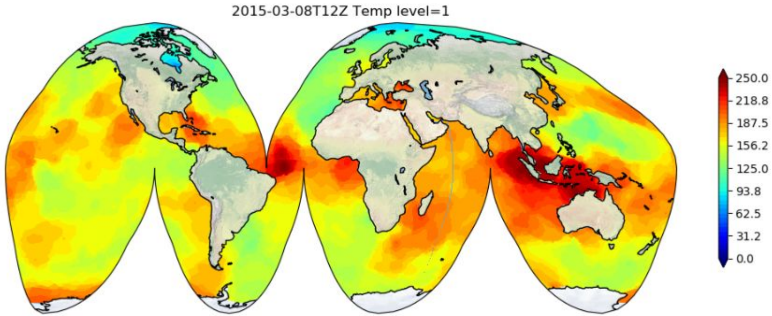
Estimated by BUMP for SSH:



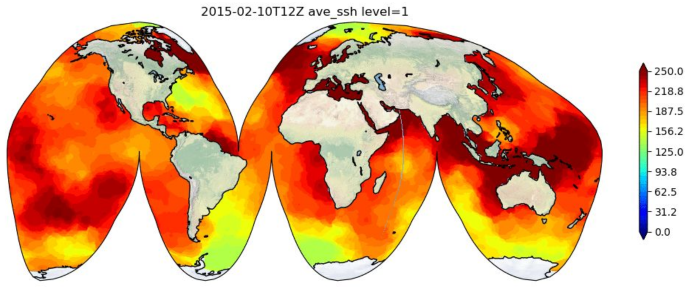
Rossby radius-based length-scale (current **SOCA** B):

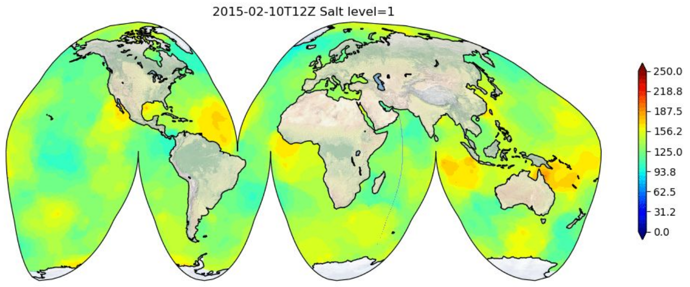


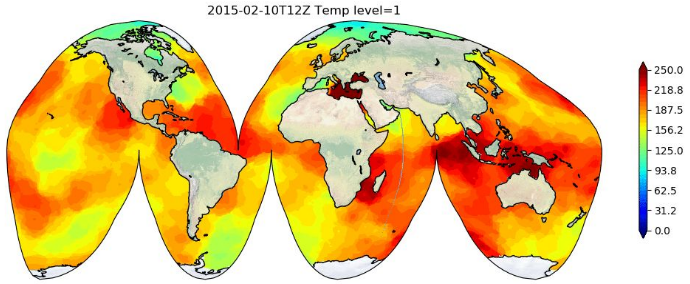
Estimated by BUMP for SST:

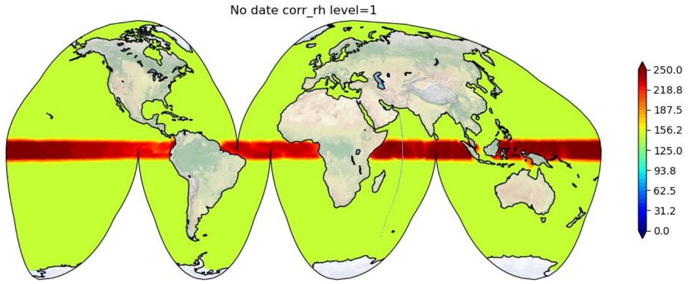


Ensemble based **B**



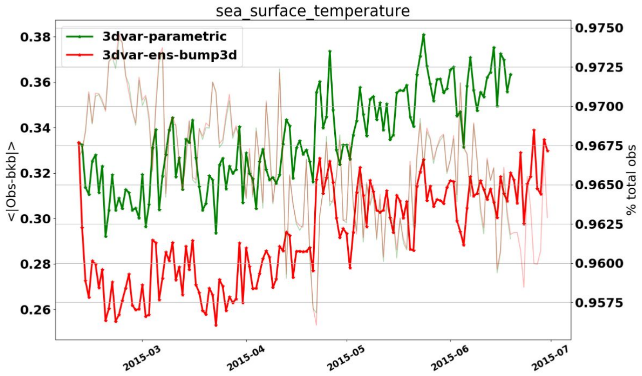




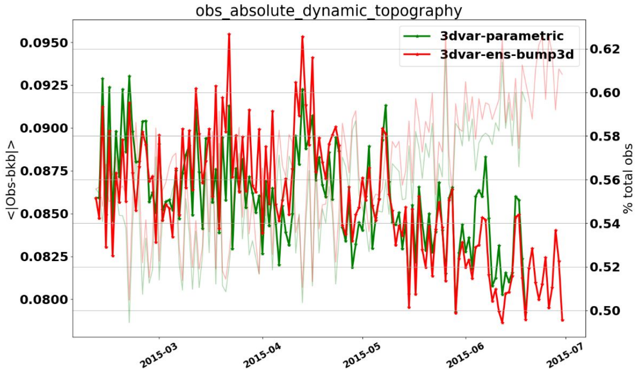


Ensemble based **B**

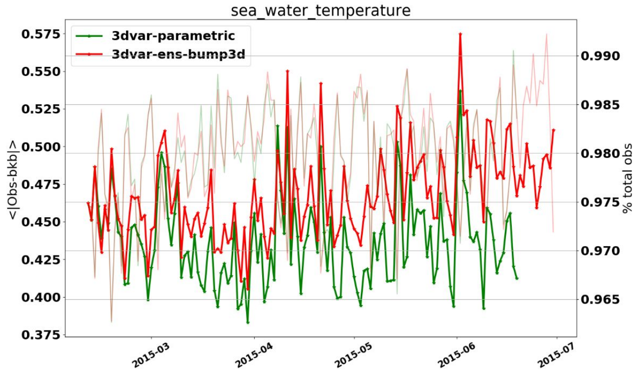
For SST: very positive impact



For absolute dynamic topography: neutral impact



For temperature: negative impact

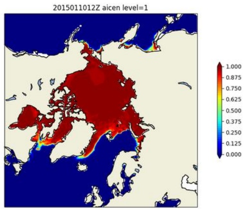


OMG!!!!! Coupled **B**

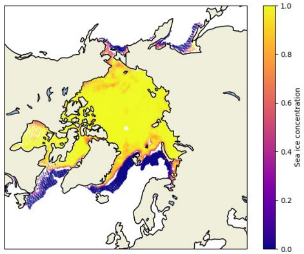
Hybrid Covariance Model

$$\boldsymbol{B} = \alpha \boldsymbol{B}_{static} + \beta \underbrace{\boldsymbol{B}_{ens}}$$

Ice concentration
Background

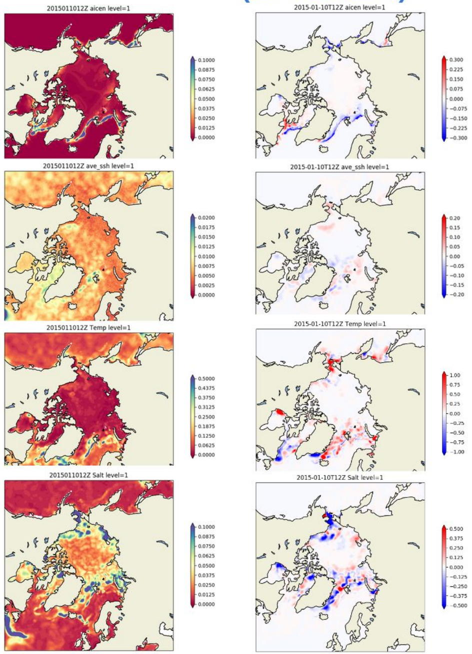


L2 Ice concentration
observations (SSMI/SSMIS)



Spread

EnVAR increment
(20 members)



Ice concentration

Sea surface height [m]

Sea surface temperature [K]

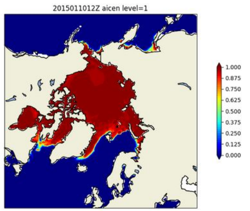
Sea surface salinity [psu]

Ensemble B provides cross-covariances between the ocean and sea ice state

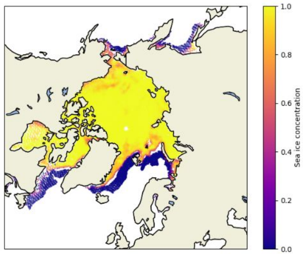
Hybrid Covariance Model

$$\boldsymbol{B} = \alpha \boldsymbol{B}_{static} + \beta \boldsymbol{B}_{ens}$$

Ice concentration Background

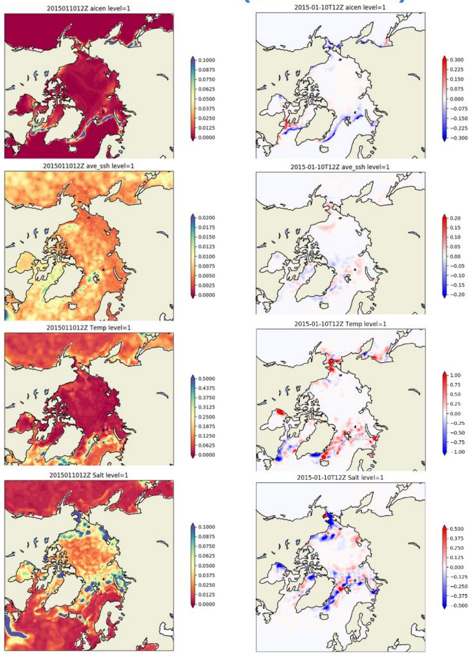


L2 Ice concentration observations (SSMI/SSMIS)



Spread

EnVAR increment (20 members)



Ice concentration

Sea surface height [m]

Sea surface temperature [K]

Sea surface salinity [psu]

Ensemble B provides cross-covariances between the ocean and sea ice state